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TEACHING MATERIAL ON

MATHEMATICS

SCHOOL OF SCIENCE

Dr. Dhrub Kumar Singh (Department Of Mathematics) ,School of Science YBN University , Ranchi

Ranchi University Question Papers - 2019

[redacted]

GROUP-B [OPTIMIZ. Meth. (CE)]

Q1 (a) (i) State matrix form of LPP

(ii) Write the difference between non-degenerate and degenerate basis feasible solution.

(iii) Write the difference between slack and surplus Variables.

$\boxed{3+3+4=10}$

(b) A company produce two types of hats A and B. Every hat A require twice as much labour time as the second hat B. If the company produce only hat B, then it can produce a total of 500 hats per day. The market limit of daily sale of hat A and B is 150 and 250 respectively. The profit on hat A and B are Rs 8 and Rs 5 respectively. Solve graphically to get max^m profit — $\boxed{10}$

Q2 (a) Use Simplex method to solve :- — $\boxed{10}$

$$\text{Min } Z = x_2 - 3x_3 + 2x_5$$

Subject to, $3x_2 - x_3 + 2x_5 \le 7$

$-2x_2 + 4x_3 \le 12$

$-4x_2 + 3x_3 + 8x_5 \le 10$, $x_1, x_2, x_3 \geq 0$

(b) Use penalty method to solve: — $\boxed{10}$

Maximize $Z = 3x_1 + 2x_2$; subject to $2x_1 + x_2 \le 2$; $3x_1 + 4x_2 \geq 12$; $x_1, x_2 \geq 0$

Q3 Solve by Big-M method — $\boxed{20}$

Maximize, $Z = x_1 + 2x_2 + 3x_3 - x_4$ subject to

$x_1 + 2x_2 + 3x_3 = 15$; $2x_1 + x_2 + 5x_3 = 20$; $x_1 + 2x_2 + 3x_3 + x_4 = 10$.

Q4 (a) Solve by two-phase simplex method :- — $\boxed{14}$

Maximize, $Z = -4x_1 - 3x_2 - 9x_3$ Subject to

$2x_1 + 4x_2 + 6x_3 \geq 15$; $6x_1 + x_2 + 6x_3 \geq 12$; $x_1, x_2, x_3 \geq 0$.

(b) Find the dual of the following LPP — $\boxed{06}$

Maximize, $Z = 3x_1 - x_2 + x_3$ subject to

$4x_1 - x_2 \le 8$; $5x_1 - 6x_3 \le 13$; $8x_1 + x_2 + 3x_3 \geq 2$, $x_1, x_2, x_3 \geq 0$

Q5. Solve the following transportation problem by MODI method with initial solution by VAM — $\boxed{20}$

	D_1	D_2	D_3	D_4	Supply
O_1	2	2	2	1	3
O_2	10	8	5	4	7
O_3	7	6	6	8	5
Demand	4	3	4	4	15

Q6 (a) (i) Distinguish between transportation and assignment problem.

(ii) Give a mathematical formulation of assignment problem. $\boxed{5+5=10}$

(b) A company has 4 machines to do 3 jobs. Each job can be assigned to only one machine. The cost of each job on each machine is given below. Determine the job assignments that will minimize the total cost: — $\boxed{10}$

MACHINE

$$\text{JOB}\quad \begin{array}{c} A \\ B \\ C \end{array}\begin{bmatrix} 18 & 24 & 28 & 32 \\ 8 & 13 & 17 & 18 \\ 10 & 15 & 19 & 22 \end{bmatrix}$$

Q7. Use the Wolfe's method to solve the QPP:— $\boxed{20}$

Maximize, $Z = 2x_1 + x_2 - x_1^2$ subject to

$2x_1 + 3x_2 \leq 6$; $2x_1 + x_2 + s_2^2 = 4$; $x_1, x_2 \geq 0$

Q8 (a) Write the Dynamic Programming Algorithm $\boxed{4}$

(b) Distinguish between Linear and Dynamic programming. $\boxed{6}$

(c) Maximize, $z = y_1^2 + y_2^2 + y_3^2$

Subject to, $y_1 + y_2 + y_3 \geq 15$

$y_1, y_2, y_3 \geq 0$

—— X —— X —— X ——

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Optimization Method (Civil)

Q1(a)(i) State matrix form of LPP.

Soln An example will best illustrate how to express the LPP in a matrix form

Let us consider the following LPP :-

Maximize $z = 4x_1 + 2x_2 + 6x_3$ Subject to

$$2x_1 + 3x_2 + 2x_3 \geq 0$$
$$3x_1 + 4x_2 = 8$$
$$6x_1 - 4x_2 + x_3 \leq 10$$

and $x_1, x_2, x_3 \geq 0$

The standard matrix form of this problem is

Maximize $z = (4, 2, 6, 0, 0)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$ Subject to

$$\begin{bmatrix} 2 & 3 & 2 & -1 & 0 \\ 3 & 4 & 0 & 0 & 0 \\ 6 & -4 & 1 & 0 & 1 \end{bmatrix}_{3\times5} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}_{5\times1} = \begin{bmatrix} 6 \\ 8 \\ 10 \end{bmatrix}_{3\times1}$$

Q1(a)(ii) Write the difference between non-degenerate and degenerate basis feasible solution.

Ans: Non-Degenerate Basic Feasible Solution:-

A basic feasible solution is said to be non-degenerate if it has exactly m positive (non-zero) x_j. The solution on the other hand is degenerate if one or more of the m basic variables vanish.

Degenerate means the problem will not generate new solution in further iterations.

~~[illegible]~~ If we consider that there are m equality constraints and $(m+n)$ is the no. of variables $(m \leq n)$ and a start for the optimal solution is made by putting n unknown (out of $m+n$ unknowns) equal to zero and then solving for m equations in remaining m unknowns, provided the solution exist and is unique. The n-zero variables are called non-basic variables and the remaining m variables are called basic variables which form a basic solution.

Hence a Basic feasible solution in the above if the solution yield all non-negative basic variables, it is called basic feasible solution otherwise, it is infeasible i.e, solution having variable less than zero.

Feasible solution: x_j $(j = 1, 2, 3, \cdots, n)$ is a feasible solution of the LPP if it satisfies

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n \; (\leq, =, \geq) \; b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n \; (\leq, =, \geq) \; b_2 \\ \cdots \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n \; (\leq, =, \geq) \; b_m \end{array}\right\} \quad (1)$$

and $x_j \geq 0$ where $j = 1, 2, 3, \cdots n$ — (2)

taking both equations (1) & (2) in view.

Q1 (a) (iii) Write the difference between slack and surplus variable.

Solution. Slack and Surplus variables :-

The general LPP can be expressed as follows—
Find the values of the decision variables $x_1, x_2, x_3 \cdots, x_n$ which satisfies the above Constraints ① & ② explained in Q1(a)(ii), or minimize the objective function (profit, loss, cost etc) which is a linear function of x_j such as $Z = c_1x_1 + c_2x_2 + c_3x_3 + \cdots + c_nx_n$ (obj fn)

The constants c_j ($j = 1, 2 \cdots n$) in eqn above are called cost coefficients

The constants b_i ($i = 1, 2, 3, \cdots, n$) in the constraints conditions are called stipulations.

The constants a_{ij} ($i = 1, 2, \ldots m$ and $j = 1, 2, 3 \cdots n$) are called structural coefficients.

(a)(i) In matrix notation, the problem can be written as

Maximize (or minimize) $Z = cX$ subject to

$$AX (\le, =, \ge) b \; ; \; X \ge 0, \; b \ge 0$$

(ii) Ex: To express the following canonical form to LPP into standard form :-

Maximize $Z = 3x_1 + 2x_2$ subject to

$2x_1 + x_2 \le 2$

$3x_1 + 4x_2 \ge 12$ and $x_1, x_2 \ge 0$

We express it as

Maximize $Z = 3x_1 + 2x_2$ subject to

$$2x_1 + x_2 + s_1 = 2$$
$$3x_1 + 4x_2 - s_2 = 12 \text{ and } x_1, x_2, s_1, s_2 \ge 0$$

Clearly we have introduced in the given example a slack (s_1) variable and a surplus (s_2) variable to write the canonical form in the standard form.

Thus we can define slack and surplus variable as follows:—

In the given canonical form of a general LPP, the inequality constraints are changed into equality constraints by adding non-negative variables called slack variables or slacks if the constraints are $\leq$ and by subtracting non-negative variables called surplus variables if the constraints are $\geq$ from the left-hand sides of such constraints. Thus the general LPP can be expressed in the standard form as

$$Z = CX \text{ subject to}$$

$$AX = b;\ X \geq 0,\ b \geq 0$$

(b) A Company produce two types of hates A and B. Every hates A require twice as much labour time as the second hat B. If the company produce only hat B, then it can produce a total of 500 hats per day. The market limit of daily sale of hat A and B is 150 and 250 respectively. The profit on hat A and B are Rs 8 and Rs 5 respectively. Solve graphically to get maximum profit.

Soln Let x_1 and x_2 represents the number of units produce of hat A and B respectively. the total profit Z, would be equal to

ie, $Z = 8x_1 + 5x_2$ (obj. fn)

~~Sub to $2x_1 + x_2$~~

Since the company can produce at the most 500 hats in a day and A type of hats require twice as much time as that of type B, production restriction is given by

	A	B
Lab. time	2hr	1hr
Prod.	X	500/day
Market lmt.	150	250
Profit	8	5

$2tx_1 + tx_2 \le 500t$, where t is the labour time per unit of second type ie,

$$2x_1 + x_2 \le 500$$

But

$$x_1 \le 150$$
$$x_2 \le 250$$

} as the limitation of the sales

$$x_1 \geq 0$$
$$x_2 \geq$$

} as non-negative production

Thus the problem can be finally written as

To find x_1 and x_2 to Maximize $Z = 8x_1 + 5x_2$ subject to

$2x_1 + x_2 \le 500$; $x_1 \le 150$; $x_2 \le 250$; $x_1 \geq 0$; $x_2 \geq 0$.

Graphical solution :- Clearly any point (x_1, x_2) in the positive quadrant will certainly satisfy non-negativity restrictions; $x_1 \geq 0$; $x_2 \geq 0$

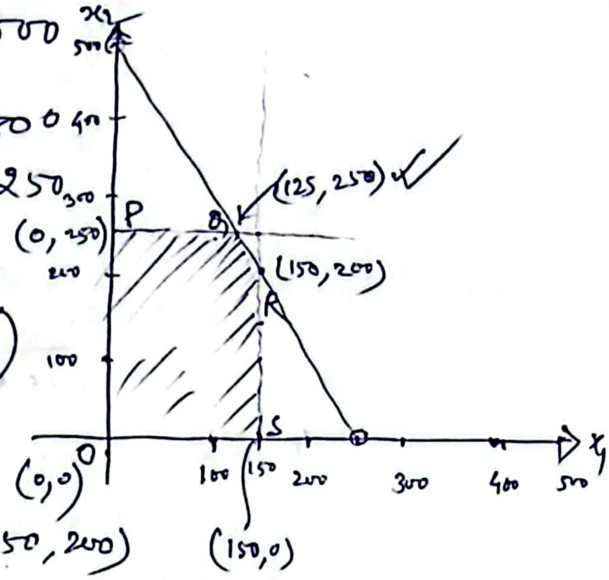
Now to plot $2x_1 + x_2 \leq 500$

we put $x_1 = 0$ to find $x_2 = 500$

and put $x_2 = 0$ to find $x_1 = 250$

also for $x_1 \leq 150$ and $x_2 = 250$,

the feasible region (polygon) obtained as OPQRSO the vertices of the point



O(0,0), P(0,250), Q(125,250), R(150,200)

S(150,0) . This polygon will give the optimum value of Z and is always unique but there will be infinite ~~value of Z~~ number of feasible solutions which give unique value of Z. Thus the two points or corners Q(125, 250) and R(150,200) as well as any other point on the line AB (segment) will give optimal solution of this problem.

It should be ~~noted~~ noted that if a LPP has more than one optimum solution, there exists alternative optimum solutions.

And one of the optimum solutions will be corresponding to corner point Q i.e. Q(125, 250) is with maximum profit Z = Rs 2,250.

82(a) Use Simplex method to solve

$$\text{Min } z = x_2 - 3x_3 + 2x_5$$

Subject to, $3x_2 - x_3 + 2x_5 \leq 7$

$-2x_2 + 4x_3 \leq 12$

$-4x_2 + 3x_3 + 8x_5 \leq 10$

and $x_1, x_2, x_3 \geq 0$

Solⁿ $\boxed{\text{The question given is wrong}}$.

It can be corrected as follows:—

$\text{Min } z = x_1 - 3x_2 + 2x_3$ subject to

$3x_1 - x_2 + 3x_3 \leq 7,$

$-2x_1 + 4x_2 \leq 12,$

$-4x_1 + 3x_2 + 8x_3 \leq 10$ and $x_1, x_2, x_3 \geq 0$

Solⁿ This is the problem of minimization. Converting the objective function from minimization to maximization, we have

$$\text{Max } -z = -x_1 + 3x_2 - 2x_3$$

or $\text{Max } z' = -x_1 + 3x_2 - 2x_3$ where $z' = -z$.

Here we can write the solution in a single table. But the students should verify them making individual tables and this is advised, as such.

Soln.

Simplex Table

Basic variable	C_B	X_B	$c_j \to$ -1 x_1	3 x_2	-2 x_3	0 x_4	0 x_5	0 x_6	Min. Ratio X_B/X_K
x_4	0	4	1	-1	1	1	1	1	—
$\leftarrow x_5$	0	2	0	$\boxed{4}$	0	0	1	0	$12/4 \leftarrow$ min
x_6	0	4	8	3	8	0	0	0	$10/3$
$x_1 = x_2 = x_3 = 0$	$z' = 0, z = 0$		1 ↑	-3^* ↑	2	0 ↓	0	0	$\leftarrow \Delta_j$
$\leftarrow x_4$	0	10	$\boxed{5/2}$	0	3	1	1/4	0	$\frac{10}{5/2} \leftarrow$
$\to x_2$	0	10	-1/2	1	0	0	1/4	0	—
x_6	0	1	5/2	0	8	0	-3/4	0	—
$x_3 = x_4 = x_2 = 0$	$z' = 11$ $\therefore z = -11$		0	0	13/5	1/5	8/10	0	$\leftarrow \Delta_j \geq 0$

Thus the optical solution is $x_1 = 4, x_2 = 5, x_3 = 0$ & Min $z = -11$ Ans.

Q2(b) Use Penality method to solve

Maximize $z = 3x_1 + 2x_2$

Subject to $2x_1 + x_2 \leq 2$

$3x_1 + 4x_2 \geq 12$ and $x_1, x_2 \geq 0$

Soln. Introducing slack and surplus variables then our problem becomes

$$Z = 3x_1 + 2x_2 + 0S_1 + 0S_2 - MA_1$$

Subject to $2x_1 + x_2 + S_1 = 2$, (S_1 = slack variable)

$3x_1 + 4x_2 - S_2 + A_1 = 12$ (S_2 = surplus variable)

(A_1 = Artificial variable)

Now setting $x_1 = x_2 = S_2 = 0$ we get $S_1 = 2$ and $A_1 = 12$ as the BFS.

Now let us ~~write~~ create Table-I Setting $s_1 = 1$ and $A_1 = 4$ for our simplicity we have

Tableau - I

C_j		3	2	0	0	-M		
C_i	Basis	x_1	x_2	s_1	s_2	A_1	b_0	θ_i = Min R
0	S_1	2	1	1	0	0	1	
-M	A_1	3	4	0	-1	1	4	

Tableau - II

$C_j \rightarrow$		3	2	0	0	-M		
C_i	Basis	x_1	x_2	S_1	S_2	A_1	b_0	θ_i = Min b_0/x_{i0}
0	S_1	2	1	1	0	0	1	1 tied
-M	A_1	3	$\boxed{4}$	0	-1	1	4	1 → Min
	Z_j	-3M	-4M	0	M	-M		
	$C_j - Z_j$	3+3M	2+4M	0	-M	0		

$\uparrow$ in Max $\qquad$ $\downarrow$ out

Key element = 4

* Observing θ_i we find that both the rows have equal θ, i.e, 1. So Arbitrary selection of one of these variables may result in one or more variables becoming zero in the next iteration and the problem is said to be degenerate, i.e, not generating new solution in

the next iteration. This means that the subsequent iterations may not produce improvement in the value of the objective function. As a result it is possible to repeat the same sequence of simplex iterations endlessly without improvement called "cycling or circling or looping. The difficulty may be overcome by applying the Perturbation Method (by A. Charnes) whose procedure is given below:-
Fortunately the problem in which cycling occurs are very rare. So we in this method first of all

(i) divide each element in the tied rows by the positive coefficients of the key column in that row & next
(ii) compare the resulting ratios column by column, first in the identity matrix and then in the body matrix from left to right and
(iii) The row which first ~~column~~ contains the smallest algebraical ratio contains the outgoing variable

Actually when there is a tie between a slack and artificial variables to leave the basis, the preference shall be given to artificial variable to leave the basis and there is no need to apply the Perturbation method in such cases.

Applying the above perturbation rule we have we have (we perturb the identity matrix to break the tie)

Identity Matrix

	S_1	S_2	A_1
1st row	$\frac{1}{1}=1$	$\frac{0}{1}=0$	$\frac{0}{1}=0$
2nd row	$\frac{0}{4}=0$	negative	$\frac{1}{4}=0.25$

We observe that the the 2nd row contains first the smallest algebraic ratio of 0, hence, this will be the key row.

Now replacing A_1 by x_2 and make tableau IIIrd.

Entering x_2, ~~and~~ exiting A_1 from basis and droping A_1 from column too we on applying $R_2(\text{new}) \to R_2(\text{old})/4$ we have,

Tableau - III

C_j	Basis	x_1	x_2	S_1	S_2	b_0
0	S_1	2	1	1	0	1
2	x_2	3/4	1	0	$-1/4$	1

Again we make Tableau - IV by applying $R_1(\text{new}) \to R_1(\text{old}) - 1 \times R_2(\text{new})$

Tableau-IV

	$C_j \to$	3	2	0	0		
C_j	Basis	x_1	x_2	S_1	S_2	b_0	θ_i
0	S_1	$\boxed{5/4}$	0	1	$1/4$	$0=\epsilon$	$\frac{4}{5}\epsilon \to$ Min
2	x_2	$3/4$	1	0	$-1/4$	1	$\frac{4}{3}$
	Z_j	$3/2$	2	0	$-1/2$	2	
	$C_j - Z_j$	$3/2$	0	0	$1/2$		

↑ in Max   ↓ out (ϵ is the least +ve ratio, $\epsilon \to 0$)   key element $= \boxed{5/4}$

Again creating Tableau-V by applying $R_1(new) \to R_1(old)/(5/4)$ we get

Tableau-V

C_j	Basis	x_1	x_2	S_1	S_2	b_0
0	S_1	1	0	$4/5$	$1/5$	$\frac{4}{5}\epsilon$
2	x_2	$3/4$	1	0	$-1/4$	1

Again forming Tableau-VI by applying $R_2(new) \to R_2(old) - \frac{3}{4} \times R_1(new)$, we get

Tableau-VI

C_j	Basis	x_1	x_2	S_1	S_2	b_0	θ_i
3	x_1	1	0	$4/5$	$\boxed{1/5}$	$\frac{4}{5}\epsilon$	$4 \to$
2	x_2	0	1	$-3/5$	$-2/5$	$1-\frac{3}{5}\epsilon$	$-\frac{5}{2}+\frac{3}{2}\epsilon$
	Z_j	3	2	$6/5$	$-1/5$	2	
	$C_j - Z_j$	0	0	$-6/5$	$1/5$		

key element → $1/5$

in ↓   out ↑ max   "x_1 replaces S_1"

Tableau-V

$C_j \rightarrow$		3	2	0	0		
C_j	Basis	x_1	x_2	s_1	s_2	b_0	θ_i
0	s_1	$\boxed{5/4}$	0	1	1/4	$0 = \epsilon$	$\frac{4}{5}\epsilon \rightarrow$ min
2	x_2	3/4	1	0	$-1/4$	1	$\frac{4}{3}$
	Z_j	3/2	2	0	$-1/2$	2	
	$C_j - Z_j$	3/2	0	0	1/2		

$\uparrow$ in Max $\downarrow$ out (ϵ is the least +ve ratio, $\epsilon \rightarrow 0$) key element $= \boxed{5/4}$

Again creating Tableau-V by applying $R_1(new) \rightarrow R_1(old)/(5/4)$ we get

Tableau-V

C_j	Basis	x_1	x_2	s_1	s_2	b_0
0	s_1	1	0	4/5	1/5	$\frac{4}{5}\epsilon$
2	x_2	3/4	1	0	$-1/4$	1

Again forming Tableau-VI by applying $R_2(new) \rightarrow R_2(old) - \frac{3}{4} \times R_1(new)$, we get

Tableau-VI

C_j	Basis	x_1	x_2	s_1	s_2	b_0	θ_i
3	x_1	1	0	4/5	$\boxed{1/5}$	$\frac{4}{5}\epsilon$	4 $\rightarrow$
2	x_2	0	1	$-3/5$	$-2/5$	$1-\frac{3}{5}\epsilon$	$-\frac{5}{2}+\frac{3}{2}\epsilon$
	Z_j	3	2	6/5	$-1/5$	2	
	$C_j - Z_j$	0	0	$-6/5$	1/5		

Key element $\rightarrow$ 1/5

in $\downarrow$ out $\uparrow$ max "x_1 replaces s_1

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Creating next Tableau-VII by applying $R_1(new) \to R_1(old)/(1/5)$ we get,

Tableau VII

C_j	Basis	x_1	x_2	s_1	s_2	b_0
3	x_1	5	0	4	1	4ϵ
2	x_2	0	1	$-3/5$	$-2/5$	$1-\frac{3}{5}\epsilon$

Key element = 1

Now Making Tableau-VIII by applying

$R_2(new) \to R_2(old) - (-2/5) \times R_1(new)$

we have on entering s_2 & existing x_1 from basis basis :

Tableau-VIII

	C_j	3	2	0	0	
C_j	Basis	x_1	x_2	s_1	s_2	b_0
0	s_2	5	0	4	1	4ϵ
2	x_2	2	1	1	0	$\left(1-\frac{3\epsilon}{5}\right)+\left(\frac{2}{5}\times 4\epsilon\right) = 1+\epsilon$
Z_j		4	2	2	0	2
$C_j - Z_j$		-1	0	-2	0	

The optimal solution is

$x_1 = 0,\ x_2 = 1,\ Z_{max} = 2$

$s_1 = 0,\ s_2 = 0,\ A_1 = 0$ Ans

Q3. Solve by Big-M Method

Maximize, $Z = x_1 + 2x_2 + 3x_3 - x_4$

Subject to,
$$x_1 + 2x_2 + 3x_3 = 15$$
$$2x_1 + x_2 + 5x_3 = 20$$
$$x_1 + 2x_2 + 3x_3 + x_4 = 10.$$
and $x_1, x_2, x_3, x_4 \geq 0.$

Soln.

We find here that all b_i are non-negative. So, introducing artificial variables A_1, A_2, A_3, we get
$$x_1 + 2x_2 + 3x_3 + A_1 = 15$$
$$2x_1 + x_2 + 5x_3 + A_2 = 20$$
$$x_1 + 2x_2 + x_3 + x_4 + A_3 = 10$$
where $x_1, x_2, x_3, x_4, A_1, A_2, A_3$ are all ≥ 0.

Now artificial variables with their values greater than '0' disturb the equality required by the general LP model. Hence A_1, A_2, A_3 must not appear in the final solution. To get this, these artificial variables are assigned a large penalty (a large negative value $-M$) in the objective f^n Z. Now we have

$$Z = x_1 + 2x_2 + 3x_3 - x_4 - MA_1 - MA_2 - MA_3$$

Putting $x_1 = 0, x_2 = 0, x_3 = 0$ and $x_4 = 0$ in the equations of constraints we obtain $A_1 = 15, A_2 = 20, A_3 = 10$ & this is the initial BSF solution to the artificial system. Putting it in Matrix form we have the tableau I as

Tableau I (Key element)

c_j	Basis	x_1	x_2	x_3	x_4	A_1	A_2	A_3	b_0	θ_i
$-M$	A_1	1	2	3	0	1	0	0	15	5
$-M$	A_2	2	1	$\boxed{5}$	0	0	1	0	20	4 → Min
$-M$	A_3	1	2	1	1	0	0	1	10	10
	z_j	$-4M$	$-5M$	$-9M$	$-M$	$-M$	$-M$	$-M$	initial solution	
	$c_j - z_j$	$1+4M$	$2+5M$	$3+9M$	$-1+M$	0	0	0		

Let us make table-II (374) applying the following
$R_2(new) \rightarrow R_2(old)/5$

Tableau-II

C_j	Basis	x_1	x_2	x_3	x_4	A_1	A_2	A_3	b_0
$-M$	A_1	1	2	3	0	1	0	0	15
$-M$	A_2	2/5	1/5	1	0	0	1/5	0	4
$-M$	A_3	1	2	1	1	0	0	1	10

Again forming Tableau-III by applying the following:
$R_1(new) \longrightarrow R_1(old) - 3 \times R_2(new)$ and
$R_3(new) \rightarrow 3 \times R_3(old) - 1 \cdot R_2(new)$. Also entering x_3, exiting A_2 and droping A_2, the artificial variable from the column ~~also~~, we get Tableau III as

Tableau-III

	$C_j \rightarrow$	1	2	3	-1	$-M$	$-M$		
C_j	Basis	x_1	x_2	x_3	x_4	A_1	A_3	b_0	θ_i Min Ratio
$-M$	A_1	$-1/5$	$\boxed{7/5}$	0	0	1	0	3	$15/7 \rightarrow$ Min
3	x_3	2/5	1/5	1	0	0	0	4	20
$-M$	A_3	3/5	9/5	0	1	1	1	6	10/3
	Z_j	$\frac{6-2M}{5}$	$\frac{3-16M}{5}$	3	$-M$	$-M$	$-M$		
	$C_j - Z_j$	$\frac{-1+2M}{5}$	$\frac{7+16M}{5}$	0	$-1+M$	0	0		
			in ↑ max			↓ out			

2nd soln Key elt = $\boxed{7/5}$

Next we form Tableau-IV by applying the following:
$R_1(new) \longrightarrow R_1(old)/(7/5)$

Tableau - IV

C_j	Basis $\;c_j \rightarrow$	x_1 (1)	x_2 (2)	x_3 (3)	x_4 (−1)	A_1 (−M)	A_3 (−M)	b_0
−M	A_1	−1/7	1	0	0	5/7	0	15/7
3	x_3	2/5	1/5	1	0	0	0	4
−M	A_3	3/5	9/5	0	1	0	1	6

Making next Tableau V by applying the following

$R_2(\text{new}) \rightarrow R_2(\text{old}) - \frac{1}{5} \times R_1(\text{new})$;

$R_3(\text{new}) \rightarrow R_3(\text{old}) - 9/5 \times R_1(\text{new})$

Now Exiting A_1, entering x_2 and dropping A_1 from column too we get

Tableau - V

$c_j \rightarrow$

C_j	Basis	x_1	x_2	x_3	x_4	A_3	b_0	θ_i
2	x_2	−1/7	1	0	0	0	15/7	×
3	x_3	3/7	0	1	0	0	25/7	×
−M	A_3	6/7	0	0	$\boxed{1}$	1	15/7	15/7 → min
	Z_j	$\frac{7-6M}{7}$	2	3	−M	−M		
	$C_j - Z_j$	$\frac{6M}{7}$	0	0	−1+M	0		

in ↑ max

↓ out

3rd soln Key element = 1

To make Tableau VI by applying the following

$R_1(\text{new}) \rightarrow R_2(\text{old})/1$;

$R_1(\text{new}) \rightarrow R_1(\text{old}) - 0 \cdot R_1(\text{new})$

Also Entering x_4, existing A_3 from basis and dropping A_1 from column too.

Tableau VI

$C_j \rightarrow$		1	2	3	-1		
C_j	Basis	x_1	x_2	x_3	x_4	b_0	θ_i
2	x_2	$-1/7$	1	0	0	$15/7$	-15
3	x_3	$3/7$	0	1	0	$25/7$	$25/3$
-1	x_4	$\boxed{6/7}$	0	0	1	$15/7$	$5/2 \rightarrow$ min
Z_j		$1/7$	2	3	-1		
$C_j - Z_j$		$6/7$	0	0	0		
		in ↑ Max			↓ out		

4th feasible soln

Key element = $\boxed{6/7}$

Now to construct Tableau VII by applying

$R_3(\text{new}) \rightarrow R_3(\text{old})/(6/7)$

Tableau VII

C_j	Basis	x_1	x_2	x_3	x_4	b_0
2	x_2	$-1/7$	1	0	0	$15/7$
3	x_3	$3/7$	0	1	0	$25/7$
-1	x_4	1	0	0	$7/6$	$5/2$

Key element = 1

Now constructing Tableau VIII by applying

$R_1(\text{new}) \rightarrow R_1(\text{old}) - (1/7) \times R_3(\text{new})$

$R_2(\text{new}) \rightarrow R_2(\text{old}) - (3/7) \times R_3(\text{new})$ ∴ Entering x_1, existing x_4

Tableau-VIII (377)

C_j	Basis	x_1 (1)	x_2 (2)	x_3 (3)	x_4 (−1)	b_0
2	x_2	0	1	0	1/6	5/2
3	x_3	0	0	1	−1/2	5/2
1	x_1	1	0	0	7/6	5/2
	Z_j	1	2	3	0	15
	$C_j - Z_j$	0	0	0	−1	

Since all $(C_j - Z_j) \le 0$

$\therefore$ Optimal solution is reached

Thus we have $x_1 = \frac{5}{2}$, $x_2 = \frac{5}{2}$, $x_3 = \frac{5}{2}$, $x_4 = 0$

$A_1 = 0$ $A_2 = 0$, $A_3 = 0$ and

$Z_{max} = 15$. Ans.

Q4 (a) Solve by two phase simplex method

Maximize, $Z = -4x_1 - 3x_2 - 9x_3$

Subject to, $2x_1 + 4x_2 + 6x_3 \geq 15$

$6x_1 + x_2 + 6x_3 \geq 12$

$x_1, x_2, x_3 \geq 0$

Solution :- Converting the given LPP into the standard form by introducing surplus variables S_1, S_2 and artificial variables A_1, A_2. The initial solution is given by $A_1 = 15$, $A_2 = 12$.

Phase-1 We construct an auxiliary LPP by assuming a cost 0 to all the variables and −1 to each artificial variables subject to the given set of constraints, and it is given by z^* i.e.

Maximize, $z^* = 0x_1 + 0x_2 + 0x_3 + 0S_1 + 0S_2 - 1\cdot A_1 - 1A_2$

Subject to $2x_1 + 4x_2 + 6x_3 + S_1 + A_1 = 15$

$6x_1 + x_2 + 6x_3 - S_2 + A_2 = 12$

Tableau - I

	C_j		0	0	0	0	0	-1	-1	
C_B	Basic (B)	X_B	x_1	x_2	x_3	S_1	S_2	A_1	A_2	Min Ratio $\frac{X_B}{x_3}$
-1	A_1	15	2	4	6	1	0	1	0	15/6
-1	A_2	12	6	1	6	0	-1	0	1	12/6
	z_j	-27	-8	-5	-12	1	1	-1	-1	
	$z_j - c_j$		-8	-5	-12	1	1	0	0	
-1	x_1	3	-4	3	0	-1	1	1	1	Min $\frac{X_B}{x_1}$ 3/3 = 1
0	x_3	2	1	1/6	1	0	-1/6	0	1/6	2/(1/6) = 12
	z_j	-3	4	-3	0	1	-1	-1	1	
	$z_j - c_j$			-3	0	1	-1	0	2	
0	x_2	1	-4/3	1	0	-1/3	1/3	1/3	-1/3	
0	x_3	11/6	22/18	0	1	1/18	-4/18	-1/18	4/18	
	z_j	0	0	0	0	0	0	0		
	$z_j - c_j$		0	0	0	1	1	1		

Since all $z_j - c_j \geq 0$, the current basic feasible solution is optimal. Since Max $z^* = 0$ and no artificial variable appears in the basis, we go to phase-II.

Phase-II Next we consider the final simplex table of phase-1; also consider the actual cost associated with the original variables. We then delete the artificial variables A_1, A_2 column from the table as these variables are eliminated from the basis in phase-I. So we have the following Tableau-II

Tableau-II

C_B	Basic (B)	X_B	x_1	x_2	x_3	s_1	s_2	Min $\frac{X_B}{x_1}$
	c_j		-4	-3	-9	0	0	
-3	x_2	1	$-4/3$	1	0	$-1/3$	$1/3$	—
-9	x_3	$11/6$	$22/18$	0	1	$1/18$	$-4/18$	$3/2$
	Z_j	$-\frac{39}{2}$	-7	-3	-9	$1/2$	1	
	$Z_j - C_j$		$-3 \uparrow$	0	0	$1/2$	1	
-3	x_2	3	0	1	$12/11$	$-3/11$	$-1/11$	
-4	x_1	$3/2$	1	0	$18/22$	$1/22$	$-4/22$	
	Z_j	-15	-4	-3	$-72/11$	$7/11$	1	
	$Z_j - C_j$		0	$27/11$	$7/11$	1	1	

Since all $Z_j - C_j \geq 0$, the current basic feasible solution is optimal. The optimal solution is given by Max $Z = -15$, $x_1 = 3/2$, $x_2 = 3$, $x_3 = 0$.

Q4(b) Find the dual of the LPP

Maximize, $z = 3x_1 - x_2 + x_3$

Subject to $4x_1 - x_2 \le 8$

$5x_1 - 6x_3 \le 13$

$8x_1 + x_2 + 3x_3 \ge 12$

$x_1, x_2, x_3 \ge 0$.

Solution:- Since the problem is not in the canonical form, we interchange the inequality of the second constraint.

Max $z = 3x_1 - x_2 + x_3$

Subject to $4x_1 - x_2 + 0x_3 \le 8$

$-8x_1 - x_2 - 3x_3 \le -12$

$5x_1 + 0x_2 - 6x_3 \le 13$

$x_1, x_2, x_3 \ge 0$.

i.e. Max $z = Cx$

Subject to $Ax \le B$

$x \ge 0$.

$\therefore C = (3\ -1\ 1)$ $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ $b = \begin{pmatrix} 8 \\ -12 \\ 13 \end{pmatrix}$

$A = \begin{pmatrix} 4 & -1 & 0 \\ -8 & -1 & -3 \\ 5 & 0 & -6 \end{pmatrix}$

Dual: Let w_1, w_2, w_3 be the dual variables.

The dual problem is, Min $z' = b^T W$

Sub. to $A^T W \ge C^T$ and $W \ge 0$

i.e., Min $z' = (8\ -12\ 13) \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ subject to

Subject to $\begin{pmatrix} 4 & -8 & 5 \\ -1 & -1 & 0 \\ 0 & -3 & 6 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} \geq \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$

$$Z' = 8W_1 - 12W_2 + 13W_3$$

$$4W_1 - 8W_2 + 5W_3 \geq 3$$

$$-W_1 - W_2 + 0W_3 \geq -1$$

$$0W_1 - 3W_2 + 6W_3 \geq 1$$

$$W_1, W_2, W_3 \geq 0$$

Q5. Solve the following transportation problem by MODI method with initial solution by VAM:-

	D_1	D_2	D_3	D_4	Supply
O_1	2	2	2	1	
O_2	10	8	5	4	
O_3	7	6	6	8	
Demand	4	3	4	4	

Solution Since $\Sigma a_i = \Sigma b_j$, the problem is a balance TP. Therefore, there exists a feasible solution.

	D_1	D_2	D_3	D_4	Supply	P_1	P_2	P_3	P_4	P_5	P_6
O_1	2 (3)	2	2	1	3	1	—	—	—	—	—
O_2	10	8	5 (3)	4 (4)	7	1	1	3 ←	—	—	— ←
O_3	7 (1)	6 (3)	6 (1)	8	5	0	0	0	0	0	6 ←
Demand	4	3	4	4	15						
P_1	5↑	4	4	3							
P_2	3	2	1	4↑							
P_3	3	2	1	—							
P_4	7↑	6	6	—							
P_5	—	6↑	6	—							
P_6	—		6	—							

Finally the initial basic feasible solution is given as below :—

	D_1	D_2	D_3	D_4	Supply
O_1	2 (3)	2	2	1	3
O_2	10	8	5 (3)	4 (4)	7
O_3	7 (1)	6 (3)	6 (1)	8	6
Demand	4	3	4	4	16

Since the number of occupied cells $= 6 = m+n-1$ and are also independent, there exists a non-degenerate basic feasible solution.

The initial transportation cost

$$= (3\times2) + (3\times5) + (4\times4) + (1\times7)$$
$$+ (3\times6) + (1\times6) = \text{Rs } 68 \text{ Ans}$$

5. To find the optimal solution :— Applying the MODI method, we determine a set of numbers u_i and v_j for each row and column, such that $u_i + v_j = c_{ij}$ for each occupied cell. Since the 3rd row has maximum number of allocations, we give number $u_3 = 0$. The remaining numbers can be obtained as given here.

$c_{31} = u_3 + v_1 = 7 = 0 + v_1 = 7 \Rightarrow v_1 = 7$
$c_{32} = u_3 + v_2 = 6 = 0 + v_2 = 6 \Rightarrow v_2 = 6$
$c_{33} = u_3 + v_3 = 6 = 0 + v_3 = 6 \Rightarrow v_3 = 6$
$c_{23} = u_2 + v_3 = 5 = u_2 + 6 = 5 \Rightarrow u_2 = -1$
$c_{24} = u_2 + v_4 = 4 = -1 + v_4 = 4 \Rightarrow v_4 = 5$
$c_{11} = u_1 + v_1 = 2 = u_1 + 7 = 2 \Rightarrow u_1 = -5.$

We find the sum of u_i and v_j for each empty cell and enter it at the bottom left corner of the cell. Next we find the net evaluation Δ_{ij} given by the initial table as follows:–

Initial tableau

	D_1	D_2	D_3	D_4	u_i
O_1	2 (3)	2; 1, 1	2; 1, 1	1; 0, 1	$u_1 = -5$
O_2	10; 6, 4	8; 5, 3	5 (3)	4 (4)	$u_2 = -1$
O_3	7 (1)	6 (3)	6 (1)	8; 5, 3	$u_3 = 0$
	$v_1 = 7$	$v_2 = 6$	$v_3 = 6$	$v_4 = 5$	

$\therefore \Delta_{ij} = C_{ij} - (u_i + v_j)$ for each empty cell and thus we enter it at the bottom right corner of the cell.

Now since all $\Delta_{ij} > 0$ the solution is optimum and is unique. The solution is given by

$x_{11} = 3$; $x_{23} = 3$; $x_{24} = 4$

$x_{31} = 1$; $x_{32} = 3$; $x_{33} = 1$

$\therefore$ The total transportation cost is equal to

$= (3\times2) + (3\times5) + (4\times4) + (1\times7) + (3\times6) + (1\times6) = $ Rs 68 Ans

(ii) Defn. and formulation of assignment problem:-

Definition:- Let there are n jobs to be performed and n persons are available for doing these jobs. Assuming that each person can do each job at a time, though with varying degrees of efficiency. Let c_{ij} be the cost if the ith person is assigned to the jth job. The problem is to find an assignment (which job should be assigned to which person, on a one-to-one basis) so that the total cost of performing all the jobs is minimum. Problem of this kind are known as assignment problems.

An assignment problem can be stated in the form of $n \times n$ cost matrix $[c_{ij}]$ of real numbers as given in the following table

$$\begin{array}{ccc} & & \text{Jobs} \\ & & \begin{array}{ccccccc} 1 & 2 & 3 & \cdots & j & \cdots & n \end{array} \\ \text{Persons} & \begin{array}{c} 1 \\ 2 \\ 3 \\ \vdots \\ i \\ \vdots \\ n \end{array} & \begin{bmatrix} c_{11} & c_{12} & c_{13} & \cdots & c_{1j} & \cdots & c_{1n} \\ c_{21} & c_{22} & c_{23} & \cdots & c_{2j} & \cdots & c_{2n} \\ c_{31} & c_{32} & c_{33} & \cdots & c_{3j} & \cdots & c_{3n} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ c_{i1} & c_{i2} & c_{i3} & \cdots & c_{ij} & \cdots & c_{in} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ c_{n1} & c_{n2} & c_{n3} & \cdots & c_{nj} & \cdots & c_{nn} \end{bmatrix} \end{array}$$

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Q.6 a (i) Distinguish between transportation and assignment problem.

(ii) Give a mathematical formulation of assignment problem.

Soln. Distinction between Transportation and Assignment problem :-

Sl. No.	Transportation Problem	Assignment problem
1.	Number of sources and destinations need not be equal. Hence the cost matrix is not necessarily a square matrix.	1. Since assignment is done on a one-to-one basis, the number of sources and destinations are equal. Hence, the cost matrix must be a square matrix.
2.	x_{ij}, the quantity to be transported from ith origin to jth destination can take any possible value, and it satisfies the rim requirements.	x_{ij} the jth job is to be assigned to the ith person and can take either the value 1 or zero.
3.	The capacity and the requirement value is equal to a_i and b_j for the ith source and jth destination $(i=1,2,\dots,m;\ j=1,2,\dots,n)$	The capacity and the requirement value is exactly one, i.e., for each source of each destination, the capacity and the requirement value is exactly one.
4	The problem is unbalanced and total demand are not equal.	The problem is unbalanced if the cost matrix is not a square matrix.

Mathematical Formulation of an Assignment Problem

Mathematically, an assignment problem can be stated as,

$$\text{Minimize } z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}\, x_{ij}, \quad \text{where } i = 1, 2, \dots n \text{ and } j = 1, 2, \dots n$$

and subject to the restrictions

$$x_{ij} = \begin{cases} 1, & \text{if the ith person is assigned jth job} \\ 0, & \text{if not} \end{cases}$$

$$\sum_{j=1}^{n} x_{ij} = 1 \text{ (one job is done by the ith person)}$$

and $\sum_{j=1}^{n} x_{ij} = 1$ (only one person should be assigned the jth job)

where x_{ij} denotes that the jth job is to be assigned to the ith person.

(iv) A company has 4 machines to do 3 jobs. Each job can be assigned to only one machine. The cost of each job on each machine is given below. Determine the job assignments that will minimize the total cost.

Machine

$$\text{Job}\quad \begin{array}{c|cccc} & W & X & Y & Z \\ \hline A & 18 & 24 & 28 & 32 \\ B & 8 & 13 & 17 & 18 \\ C & 10 & 15 & 19 & 22 \end{array}$$

Soln: Since the cost matrix is not a square matrix, we add a dummy row D with all the elements 0.

$$\begin{array}{c|c} & \begin{matrix} W & X & Y & Z \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 18 & 24 & 28 & 32 \\ 8 & 13 & 17 & 18 \\ 10 & 15 & 19 & 20 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Now we subtract the minimum element in each row from all the elements in its row.

$$\begin{array}{c|c} & \begin{matrix} W & X & Y & Z \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 6 & 10 & 14 \\ 0 & 5 & 9 & 10 \\ 0 & 5 & 9 & 12 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Since, each element has a minimum element 0, we draw minimum number of lines to cover all zeros.

$$\begin{array}{c|c} & \begin{matrix} W & X & Y & Z \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 6 & 10 & 14 \\ 0 & 5 & 9 & 10 \\ 0 & 5 & 9 & 12 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{4\times 4} \end{array}$$

∴ The number of lines drawn to cover all zeros = 2 < the order of matrix, we form a second modified matrix.

By subtracting the ~~minimum~~ uncovered value from all other uncovered values i.e, 5 and adding 5 to the element at the point of intersection of lines ⟶

$$\begin{array}{c|cccc} & W & X & Y & Z \\ \hline A & 0 & 1 & 5 & 9 \\ B & 0 & 0 & 4 & 5 \\ C & 0 & 0 & 4 & 7 \\ D & 5 & 0 & 0 & 0 \end{array}$$

Here $N = 3 < n = 4$, the order of matrix,

Again, we subtract the smallest uncovered element from all the uncovered elements and add to the element at the point of intersection we get-

$$\begin{array}{c|cccc} & W & X & Y & Z \\ \hline A & 0 & 1 & 1 & 5 \\ B & 0 & 0 & 0 & 1 \\ C & 0 & 0 & 0 & 3 \\ D & 9 & 4 & 0 & 0 \end{array}$$

Here $N = 4 = n$. Hence, we made an assignment as follows :—

$$\begin{array}{c|cccc|}
 & W & X & Y & Z \\
A & 0 & 1 & 1 & 4 \\
B & \cancel{0} & 0 & \cancel{0} & 1 \\
C & \cancel{0} & \cancel{0} & 0 & 3 \\
D & 9 & 4 & \cancel{0} & 0
\end{array}$$

$A \rightarrow W$
$D \rightarrow Z$
$B \rightarrow X$
$C \rightarrow Y$

or

$$\begin{array}{c|cccc|}
 & W & X & Y & Z \\
A & 0 & 1 & 1 & 4 \\
B & \cancel{0} & \cancel{0} & 0 & 1 \\
C & \cancel{0} & 0 & \cancel{0} & 3 \\
D & 9 & 4 & \cancel{0} & 0
\end{array}$$

$A \rightarrow W$
$D \rightarrow Z$
$B \rightarrow Y$
$C \rightarrow X$

Since D is a dummy job, machine Z is assigned no job.

Therefore, optimum cost $= 18 + 13 + 19 =$ Rs 50/-

Q7. Use the Wolfe's method to solve the QPP:

Maximize, $Z = 2x_1 + x_2 - x_1^2$

subject to $2x_1 + 3x_2 \le 6$

$2x_1 + x_2 + s_2^2 = 4$

$x_1, x_2 \geq 0$

Soln: This problem is solved in my handout notes in the concerned chapter.

Q8 (a) Write the Dynamic Programming ~~problem~~ Algorithm.

(b) Distinguish between Linear and Dynamic programming.

(c) Maximize, $z = y_1^2 + y_2^2 + y_3^2$

Subject to $y_1 + y_2 + y_3 \geq 15$

$y_1, y_2, y_3 \geq 0$

Sol^n:- <u>Dynamic programming Algorithm</u> :-

The solution of a multistage problem by dynamic programming involves the following steps:-

Step-1: Identify the decision variables and specify the objective function to be optimized under certain limitation, if any.

Step-2: Decompose the given problem into a number of smaller subproblems. Identify the state variable at each stage.

Step-3: Write down the general recursive relationship for computing the optimal policy. Decide whether forward or backward method is to be followed to solve the problem.

Step-4: Construct appropriate stages to show the required values of the return function at each stage.

Step-5: Determine the overall optimal policy or decisions and its value at each stage. There may be more than one such optimal policies.

(b) Difference between Linear and Dynamic Programming :-

Sl. No	Dynamic Programming	Linear programming
(i)	It is a multistage decision making process that spans time intervals. However, the intervals may consist only of stages in which the problem is solved	(i) It gives a solution that will pertain only to one time period within given capacity, quantity and cost constraints.
(ii)	It is similar to callculus.	(ii) It is similar to solving sets of simultenous linear equations.
(iii)	It permits one to determine the optimal decisions for furture time periods, regardless of any earlier decisions.	(iii) It requires constant updating of values obtained, in order to reflect the current constraints necessary for an optimal answer
(iv)	It uses whatever mathematics is deemed to be appropriate for the solution of the problem	(iv) In this case, certain rules must always be followed in the iterative G.P. process
(v)	Computation technique is not easy.	(v) Computational technique is easier.

(c) Minimize $Z = Y_1^2 + Y_2^2 + Y_3^2$

Subject to $Y_1 + Y_2 + Y_3 \geq 15$

$Y_1, Y_2, Y_3 \geq 0.$

Soln:- Since the decision variables are Y_1, Y_2, Y_3, the given problem is a three stage problem defined as follows :—

CamScanner

$S_3 = Y_1 + Y_2 + Y_3 \geq 15$

$S_2 = Y_1 + Y_2 = S_3 - Y_3$

$S_1 = Y_1 = S_2 - Y_2$

$$f_3(S_3) = \min_{Y_3} \left(Y_3^2 + F_2(S_2) \right)$$

$$f_2(S_2) = \min_{Y_2} \left[Y_2^2 + F_1(S_1) \right]$$

$$f_1(S_1) = Y_1^2 = (S_2 - Y_2)^2$$

Thus $f_2(S_2) = \min_{Y_2} \left[Y_2^2 + (S_2 - Y_2)^2 \right]$

$\therefore$ $Y_2^2 + (S_2 - Y_2)^2$ is minimum, if its derivative w.r.t Y_2 is zero.

$\therefore$ $2Y_2 - 2(S_2 - Y_2) = 0$

$\therefore$ $\boxed{Y_2 = \frac{S_2}{2}}$

Hence $f_2(S_2) = \frac{S_2^2}{2}$

Now, $f_3(S_3) = \min_{Y_3} \left[Y_3^2 + f_2(S_2) \right]$

$$= \min_{Y_3} \left[Y_3^2 + \frac{1}{2}(S_3 - Y_3)^2 \right]$$

or $$f_3(15) = \min_{Y_2 \leq 15} \left[Y_3^2 + \frac{1}{2}(15 - Y_3)^2 \right]$$

Since the minimum value of the function $Y_3^2 + \frac{1}{2}(15 - Y_2)^2$ occurs for $Y_3 = 5$

$\Rightarrow f_3(15) = 5^2 + \frac{1}{2}(15-5)^2 = 75$

Thus, $s_3 = 15 \Rightarrow Y_3^* = 5$

$\therefore s_2 = s_3 - Y_3 = 15 - 5 = 10 \Rightarrow Y_2^* = \frac{s_2}{2} = 5$

$s_1 = s_2 - Y_2 = 10 - 5 = 5$

$\Rightarrow Y_1^* = s_1 = 5$

Hence, the optimal policy is

$(5, 5, 5)$ with $f_3^*(Y_3) = 75$ // Ans

* — * — * — *

The end.

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Book List

Text Books

1. Operation Research by S. D. Sharma
Kedarnath Ramnath Publication.
New Delhi.

2. Operation Research (Theory and Applications) by J. K. Sharma
MACMILLAN Publications.

3. Operation Research by A. P. Verma
S. K. Kataria & Sons Publications (Delhi)

8. Reference books:

1. Quantitative Techniques in Management.
by N. D. Vohra.